S.G. Correction E.S. 28/10/2024

Exercice 1

1) $D_f = [0;4]$ $\frac{1}{2}$

$\lim\limits_{x\to 0} \frac{f(x)-f(0)}{x-0} = \lim\limits_{x\to 0} \frac{x-\sqrt{4x-x^2}}{x} \overset{R.H.}{=} \lim\limits_{x\to 0} \frac{1-\frac{4-2x}{2\sqrt{4x-x^2}}}{1} = -\infty$ $\frac{1}{2}$

$\lim\limits_{x\to 4} \frac{f(x)-f(4)}{x-4} = \lim\limits_{x\to 4} \frac{x-\sqrt{4x-x^2}-4}{x-4} = \lim\limits_{x\to 4} \frac{1-\frac{4-2x}{2\sqrt{4x-x^2}}}{1} = -\infty$ $\frac{1}{2}$

Au point $x=0$ et $x=4$, la fonction n'est pas dérivable et admet des demi-tangentes verticales d'équations $x=0$ et $x=4$. $\frac{1}{1}$

2) $f'(x) = 1 - \frac{4-2x}{2\sqrt{4x-x^2}} = 1-\frac{2-x}{\sqrt{4x-x^2}} = \frac{\sqrt{4x-x^2}-(2-x)}{\sqrt{4x-x^2}}$

$f'(x) \geq 0 : \sqrt{4x-x^2} \geq 2-x$

si $2-x \geq 0$, $-x \geq -2$, $x \leq 2$

$\begin{cases} 4x-x^2 \geq 4-4x+x^2 \\ -2x^2+8x-4 \geq 0 \end{cases}$

$\Delta = 64-32 = 32$

$x_1 = \frac{-8-\sqrt{32}}{-4} = \frac{-8-4\sqrt{2}}{-4} = 2-\sqrt{2}$

$x_2 = 2+\sqrt{2}$

x	0		$2-\sqrt{2}$		$2+\sqrt{2}$	
		$-$	0	$+$	0	$-$

alors $x \in [0, 2-\sqrt{2}]$; $f(x) \leq 0$

$x \in [2-\sqrt{2}; 0]$; $f(x) \geq 0$

si $2-x \leq 0$, $-x \leq -2$, $x \geq 2$

$f(x) \geq 0$: $\sqrt{4x-x^2} \geq 2-x$ tj vrai

$x \in [2,4]$; $f(x) \geq 0$

x	0		$2-\sqrt{2}$		2		4
$f'(x)$	∞	$-$	0	$+$		$+$	∞
$f(x)$	0	↘	$-0{,}83$	↗	0	↗	4

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3) $g(2a-x) + f(x) = 2b$?

$g(4-x) + f(x) =$

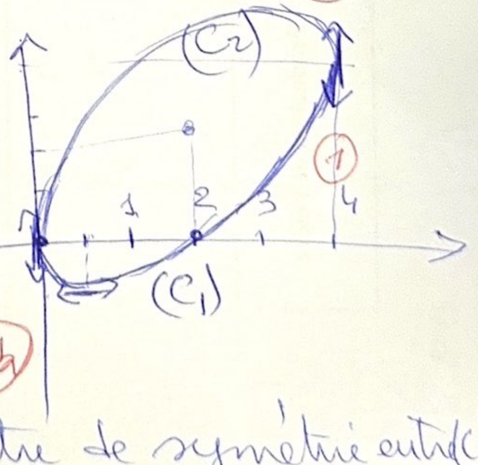
$4-x+\sqrt{4(4-x)-(4-x)^2} + x - \sqrt{4x-x^2}$

$= 4-x+\sqrt{16-4x-16+8x-x^2} + x - \sqrt{4x-x^2}$

$= 4 + \sqrt{4x-x^2} - \sqrt{4x-x^2}$

$= 4$ alors $I(2,2)$ est le centre de symétrie entre (C_1) et (C_2)

$1\frac{1}{2}$



4) $2x^2+y^2-2xy-4x=0$

$x^2+y^2-2xy+x^2-4x=0$

$(x-y)^2=4x-x^2$

$(x-y)=\sqrt{4x-x^2}$ ou $-\sqrt{4x-x^2}$

$x-\sqrt{4x-x^2}=y$ (C_1)

ou

$x+\sqrt{4x-x^2}=y$ (C_2)

$1\frac{1}{2}$

Exercice 2

$1\frac{1}{2}$

A. $\dfrac{4+4i-1-1+2i+1}{4-4i-1-1-2i+1}=\dfrac{3+6i}{3-6i}=\dfrac{9+36i-36}{9+36}=\dfrac{-27+36i}{45}$

$=\dfrac{-3}{5}+\dfrac{4i}{5}$

B. $1-iz=z+i+iz-1$

$-iz-z-iz=i-1-1$

$z(-1-2i)=i-2$

$z=\dfrac{(i-2)(-1+2i)}{(-1-2i)(-1+2i)}=\dfrac{-i-2+2-4i}{1+4}$

$1\frac{1}{2}$

$z=\dfrac{-5i}{5}=-i$

C. 1) $z'=\dfrac{2+i-i}{i(2-i)-1}=\dfrac{2}{2i-i^2-1}=\dfrac{2}{2i}=\dfrac{2i}{2i^2}=-i$

$z'=1e^{-\frac{\pi}{2}i}$ $1\frac{1}{2}$

2) a) $x'+iy'=\dfrac{x+iy-i}{i(x-iy)-1}=\dfrac{x+iy-i}{ix+y-1}=\dfrac{(x+iy-i)(y-1-ix)}{(y-1+ix)(y-1-ix)}$

$x'+iy'=\dfrac{\cancel{xy}-x-ix^2+iy^2-iy+\cancel{xy}-iy+i-x}{(y-1)^2+x^2}$

$x'+iy'=\dfrac{2xy-2x}{(y-1)^2+x^2}+i\dfrac{(y^2-x^2-2y+1)}{(y-1)^2+x^2}$ 1

$x'=\dfrac{2x(y-1)}{(y-1)^2+x^2}$ 1 et $y'=\dfrac{(y-1)^2-x^2}{(y-1)^2+x^2}$

b) $y=x+1$; $x'=\dfrac{2x(x+1-1)}{(x+1-1)^2+x^2}+i\dfrac{(x+1-1)^2-x^2}{(x+1-1)^2+x^2}$

$z'=\dfrac{2x^2}{2x^2}+i\dfrac{x^2-x^2}{2x^2}\longrightarrow z'=1$ $1\frac{1}{2}$

3) a) $z+z'=z+\dfrac{z-i}{i\bar{z}-1}=\dfrac{zi\bar{z}-z+z-i}{i\bar{z}-1}=\dfrac{zi\bar{z}-i}{i\bar{z}-1}$

$z+z'=\dfrac{i(z\bar{z}-1)}{i\bar{z}-1}$ 1

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b) $M\in C(O,1)\longrightarrow|z|=1\Rightarrow z\bar{z}=1\Rightarrow z+z'=0$

alors M et M' diametrale ↓ opposés