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Exercice 1.

1) $f'(x) = 4x^3 + 3x^2 + 12x$

 $\Delta < 0 \rightarrow f''(x)$ ne s'annule pas (B)

$f''(x) = 12x^2 + 6x + 12$

2) $f(x) = -4$; $x^3 - x^2 + 2x - 1 = -1$; $x^3 - x^2 + 2x = 0$; $x(x^2 - x + 2) = 0$

$\rightarrow x=0$ $f'(x) = 3x^2 - 2x + 2$; $(f^{-1})'(-1) = \frac{1}{f'(0)} = \frac{1}{2}$

$y = (f^{-1})'(-1)(x+1) + f^{-1}(-1)$

$y = \frac{1}{2}(x+1)$; $y = \frac{1}{2}x + \frac{1}{2}$

3) $f'(x) = 4x - 6$

$$\begin{array}{c|ccccccc} x & -3 & -2 & -1 & 0 & 1 & 2 & 3 \\ \hline f(x) & -27 & -8 & -1 & 0 & 1 & 8 & 27 \\ f'(x) & -14 & -10 & -6 & -2 & 2 & 6 & 10 \end{array}$$

$f([-3; 2]) = [-45; 36]$

4) $(g \circ f)(-1) = g(f(-1))$ $f'(-1) = g'(3) \times (-4) = -20$

5) $\lim_{x \rightarrow 5^-} \frac{x+5}{1x+5} = -1$

6) $\frac{2x^3+3}{2x^3-2x} \Big| \frac{x^2-1}{2x}$

$y = 2x$ A.D. (C)

Exercice 2. $\lim_{x \rightarrow -\infty} \sin\left(\frac{x}{2}\right) = \sin(-\pi) = 0$

$\lim_{x \rightarrow 2^-} \left(\frac{-(x+2)}{(x+2)(x-2)} - \frac{2x}{x^2-4} \right) = \lim_{x \rightarrow 2^-} \left(\frac{-3x-2}{x^2-4} \right) = +\infty$

Exercice 3.

a) $\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} \frac{\sin^2 x}{x} \stackrel{\text{R.H.}}{=} \lim_{x \rightarrow 0^+} \frac{2 \sin x \cos x}{1} = 0$

$\lim_{x \rightarrow 0^+} f(x) = 0 \rightarrow f$ continue en $x=0$ (1)

$\lim_{x \rightarrow 0} \frac{f(x) - f(0)}{x - 0} = \lim_{x \rightarrow 0} \frac{\sin^2 x}{x} = \lim_{x \rightarrow 0} \frac{\sin x}{x} \cdot \frac{\sin x}{x} = 1$

donc f dérivable en $x=0$

b) $y = f'(0)(x-0) + f(0)$

$y = x$ (1)

Données $f \circ g(2) = f(g(2)) = f\left(\frac{-3}{-1}\right) = f(3) = 4$ Paux (1)

$W'(5) = (f \circ g)'(5) = f'(g(5)) \times g'(5) = f'\left(\frac{3}{2}\right) \times \frac{1}{(5-3)^2} > 0$ Paux (2)

$g(5) = \frac{10-7}{5-3} = \frac{3}{2}$

$g'(x) = \frac{2(x-3) - 1(2x-7)}{(x-3)^2} = \frac{1}{(x-3)^2}$