SSCC Bikfaya — S2S — 27/11/2024

Corrigé du Premier Contrôle Mathématiques

Exercice I.

1) $S = -\frac{b}{a} = 2$ $\qquad P = \frac{c}{a} = -4$

$$E = \frac{x_1^2 - 3x_1x_2 + x_2^2 - 3x_1x_2}{x_2x_1 - 3x_2^2 - 3x_1^2 + 9x_1x_2}$$ (1½) (A)

$$E = \frac{S^2 - 2P - 6P}{10P - 3(S^2 - 2P)} = \frac{4+8+24}{-40-3(4+8)} = \frac{36}{-76} = -\frac{9}{19}$$

2) $\frac{5x^2-7x-3}{3x^2-2x-5} - 1 \leq 0$; $\frac{5x^2-7x-3-3x^2+2x+5}{3x^2-2x-5} \leq 0$

$\frac{2x^2-5x+2}{3x^2-2x-5} \leq 0$

$P(x) = 2x^2 - 5x + 2$ $\quad x_1 = 2 \quad x_2 = \frac{1}{2}$

$Q(x) = 3x^2 - 2x - 5$ $\quad x_1 = -1 \quad x_2 = 5/3$

x	$-\infty$		-1		$1/2$		$5/3$		2		$+\infty$
$P(x)$		+		+	0	−		−	0	+	
$Q(x)$		+	0	−		−	0	+		+	
$\frac{P(x)}{Q(x)}$		+	‖	−	0	+	‖	−	0	+	

$S =]-1; \frac{1}{2}] \cup]\frac{5}{3}; 2]$ (1½) (C)

3) $x \geq -6$ et $x \geq -1$ et $x \geq -\frac{4}{7}$ donc $x \geq -\frac{4}{7}$

$(\sqrt{x+6} + \sqrt{x+1})^2 = (\sqrt{7x+4})^2$

$x+6+x+1+2\sqrt{(x+6)(x+1)} = 7x+4$

$2\sqrt{(x+6)(x+1)} = 7x+4-2x-7$

$2\sqrt{(x+6)(x+1)} = 5x-3$ pour $x \geq 2/5$

$4(x^2+x+6x+6) = 25x^2 - 30x + 9$

$-21x^2 + 58x + 15 = 0$

$x_1 = 3$; $x_2 = -\frac{5}{21}$

or $x \geq \frac{2}{5}$

$\boxed{x = 3}$ (B) (1½)

4) $\dfrac{x^2+2x+4+mx^2-2mx+4m}{x^2-2x+4} < 0 \quad \forall x \in \mathbb{R}$

$x^2-2x+4=0$ donc $\underline{x^2-2x+4>0 \quad \forall x\in\mathbb{R}}$

$\Delta = 4-16=-12<0$

alors $(m+1)x + 2x(1-m)+4m+4 < 0 \quad \forall x \in \mathbb{R}$

$\begin{cases} \Delta < 0 \\ a<0 \end{cases}$

$\Delta = 4(1-m)^2 - 4(m+1)(4m+4)$

$\Delta = 4(m^2-2m+1) - 4(4m^2+4m+4m+4)$

$\Delta = 4m^2-8m+4-16m^2-32m-16$

$a = m+1<0$

$m<-1$

$\Delta = -12m^2-40m-12$ 2

$\Delta = 0 \to m_1 = -\frac{1}{3}\ ,\ m_2=-3$

$-\infty$	-3	-1	$-1/3$	$+\infty$
$\Delta<0$	$\leftarrow$			$\rightarrow$
$a<0$	$\leftarrow$			

$S =]-\infty ; -3[$ A

5) $g(x) = f(x+2)+1$

$g(x) = x+2+\cancel{1+}\dfrac{3}{x+2}+1 = x+\cancel{4+}\dfrac{3}{x+2}$

1 B

Exercice II.

1) $x_1=4 \quad , x_2=3 \qquad P(x)=(x-4)(x-3)$ 1

2) $Q(4) = (4-3)^n - (4-4)^{2n+1}-1 = 1-0-1=0$ 1/2 1/2

$Q(3) = (3-3)^n-(3-4)^{2n+1}-1 = 0-(-1)^{2n+1}-1 = 1-1=0$

alors $Q(x)$ est divisible par $x-4$ et par $x-3$

Donc $Q(x)$ divisible par $(x-4)(x-3)$ qui est $P(x)$ 1/2

3) $P(x)(3x^2-27)\geq 0$ et $(5x^2+2)(-x^2+1)\neq 0$

$x\neq 1$ et $x\neq -1$

x	$-\infty$	-3		3		4	$+\infty$
$P(x)$	+		+	0	−	0	+
$3x^2-27$	+	0	−	0	+		+
$P(x)(3x^2-27)$	+	0	−	0	−	0	+

2

$D_f =]-\infty;-3] \cup \{3\} \cup [4;+\infty[$

4) $g(1)=0$; $\sqrt{m-2}+\sqrt{m-1}-\sqrt{m}+\sqrt{m}-1=0$

$$\sqrt{m-2}+\sqrt{m-1}=1 \qquad m\geq 2$$
$$\sqrt{m-2}=1-\sqrt{m-1} \qquad m\geq 1$$
$$\cancel{m}-2=\cancel{1}+\cancel{m}-\cancel{1}-2\sqrt{m-1} \qquad (1\tfrac{1}{2})$$
$$2\sqrt{m-1}=2 \;;\; \sqrt{m-1}=1 \;;\; m-1=1$$
$$\boxed{m=2}$$

Exercice III.

1) 1er cas si $m-2=0$; $m=2$; $-4x+4-3=0$

$$-4x=-1$$
$$\boxed{x=1/4}$$

2ème cas si $m\neq 2$; $\Delta=(-2m)^2-4(m-2)(2m-3)$

$$\Delta=4m^2-4(2m^2-3m-4m+6)$$
$$\Delta=4m^2-8m^2+28m-24=-4m^2+28m-24$$

$m_1=1$; $m_2=6$.

m	$-\infty$	1	2	6	$+\infty$
Δ	−	0 +	‖ +	0 −	

$$S=-\frac{b}{a}=\frac{2m}{m-2}$$

m	$-\infty$	0	2	$+\infty$
S	+	0 −	‖ +	

(3) $P=\dfrac{c}{a}=\dfrac{2m-3}{m-2}$

m	$-\infty$	3/2	2	$+\infty$
P	+	0 −	‖ +	

m	$-\infty$	0	1	3/2	2	6	$+\infty$
Δ	−	−	0 +	+	‖ +	0 −	
P	+	+	+	0 −	‖ +	+	
S	+	0 −	−	−	‖ +	+	
Solutions	pas de solution dans ℝ		$x_1<x_2<0$	$x_1<0<x_2$ $\lvert x_1\rvert>x_2$	$0<x_1<x_2$	pas de solution dans ℝ	

Pour $m = 1$; $\Delta = 0$; $x_1 = x_2 = \frac{-b}{2a} = \frac{2m}{2(m-2)} = \frac{2}{-2} = -1$

pour $m = \frac{3}{2}$; $P = 0$; $x_1 = 0$; $x_2 = S = \frac{-b}{a} = \frac{2m}{m-2} = \frac{3}{-\frac{1}{2}} = 6$ (1)

pour $m = 2$; $x = \frac{1}{4}$

pour $m = 6$; $\Delta = 0$; $x_1 = x_2 = \frac{-b}{2a} = \frac{m}{m-2} = \frac{6}{6-2} = \frac{6}{4} = \frac{3}{2}$

2) pour $m \in]2 ; 6[$ $\left(\frac{1}{2}\right)$

3) $P = \frac{c}{a}$; $P = \frac{2m-3}{m-2}$; $Pm - 2P = 2m - 3$

$Pm - 2m = 2P - 3$

$m = \frac{2P-3}{P-2}$

$S = -\frac{b}{a}$; $S = \frac{2m}{m-2}$

$Sm - 2S = 2m$; $Sm - 2m = 2S$

$m = \frac{2S}{S-2}$

$m = m$; $\frac{2P-3}{P-2} = \frac{2S}{S-2}$ (2)

$2PS - 4P - 3S + 6 = 2PS - 4S$; $\boxed{-4P + S + 6 = 0}$

$\boxed{4x_1x_2 - (x_1 + x_2) - 6 = 0}$ (1)

Racines doubles : $x_1 = x_2$: $4x^2 - 2x - 6 = 0$

$x_1 = -1$ et $x_2 = \frac{6}{4} = \frac{3}{2}$

4) $\overline{AM_1} \cdot \overline{AM_2} = K$

$(x_1 - a)(x_2 - a) = K$ $A(a)$

$x_1x_2 - ax_1 - ax_2 + a^2 - K = 0$

$\boxed{4x_1x_2 - 4a(x_1 + x_2) + 4a^2 - 4K = 0}$

par identification avec (1) $-4a = -1$ $4a^2 - 4K = -6$

$\boxed{a = \frac{1}{4}}$ $\frac{4}{16} - 4K = -6$

$-4K = -6 - \frac{1}{4}$

$\boxed{K = \frac{25}{16}}$

$\left(1\frac{1}{2}\right)$

5) $BC^2 = AB^2 + AC^2 - 2AB.AC.\cos\hat{A}$; $2 = x_1^2 + x_2^2 - 2x_1x_2\cos 60$

$x_1^2 + x_2^2 - 2x_1x_2\left(\frac{1}{2}\right) = 2$; $S^2 - 2P - P - 2 = 0$

$\left(\frac{2m}{m-2}\right)^2 - 3\left(\frac{2m-3}{m-2}\right) - 2 = 0$; $4m^2 - (6m-9)(m-2) - 2(m-2)^2$

$4m^2 - 6m^2 + 12m + 9m - 18 - 2m^2 + 8m - 8 = 0$ (1½)

$-4m^2 + 29m - 26 = 0$

$\boxed{m_1 = \frac{29 - 5\sqrt{17}}{8} \text{ accept}}$ et $m_2 = \frac{29 + 5\sqrt{17}}{8}$ inaccept

Exercice IV

1) $x^2 + 2x(2m+1) + y^2 + 2(m-3)y + 5 - 2m = 0$

$(x + (2m+1))^2 - (2m+1)^2 + (y + (m-3))^2 - (m-3)^2 + 5 - 2m = 0$

$(x + (2m+1))^2 + (y + (m-3))^2 = (2m+1)^2 + (m-3)^2 - 5 + 2m$

$(x + (2m+1))^2 + (y + (m-3))^2 = 4m^2 + 4m + 1 + m^2 - 6m + 9 - 5 + 2m$

$(x + (2m+1))^2 + (y + (m-3))^2 = 5m^2 + 5$ (1½)

$R^2 = 5m^2 + 5 > 0$ pour tout $m \in \mathbb{R}$

alors le cercle (C_m) existent pour tout $m \in \mathbb{R}$

3) (C_m) tangent à $y + 1 = 0$ si $d(I_m, (d)) = R_m$

2) $I_m(-2m-1, -m+3)$ (½) : $\frac{|0 + (-m+3)(1) + 1|}{\sqrt{0+1}} = \sqrt{5m^2+5}$

$\frac{|-m+3+1|}{\sqrt{1}} = \sqrt{5m^2+5}$; $|-m+4| = \sqrt{5m^2+5}$ (1½)

$m^2 - 8m + 16 = 5m^2 + 5$; $-4m^2 - 8m + 11 = 0$

$m_1 = \frac{-2+\sqrt{15}}{2}$; $m_2 = \frac{-2-\sqrt{15}}{2}$

4) $\begin{cases} x = -2m - 1 \\ y = -m + 3 \end{cases}$ x-2

$$\begin{cases} x = -2m - 1 \\ -2y = 2m - 6 \end{cases}$$

$x - 2y = -7$ → $\boxed{x - 2y + 7 = 0}$ c'est le lieu des points I. (1)

5) $d(I,(x'Ox)) < R$; $\frac{|-m+3|}{1} < \sqrt{5m^2+5}$

$(x'Ox)$: $y = 0$

$m^2 - 6m + 9 - 5m^2 - 5 < 0$

$-4m^2 - 6m + 4 < 0$

$m_1 = \frac{1}{2}$; $m_2 = -2$

m	$-\infty$		-2		$\frac{1}{2}$		$+\infty$
$-4m^2-6m+4$		$-$	0	$+$	0	$-$	

pour $m < -2$ ou $m > \frac{1}{2}$

(1)

$y = 0$: $x^2 + 2x(2m+1) + 5 - 2m = 0$

$O \in [AB]$ donc $P < 0$ alors $5 - 2m < 0$

$-2m < -5 \to m > \frac{5}{2}$ $\boxed{\text{Donc } S =]\frac{5}{2}, +\infty[}$

6) Centre du cercle : $x_I = \frac{x_A + x_B}{2} = \frac{S}{2} = \frac{-2(2m+1)}{2}$

$x_I = -2m - 1$; $y_I = 0$

$I(-2m-1 ; 0)$

$AB = |x_B - x_A| \to AB^2 = (x_B - x_A)^2 = x_A^2 + x_B^2 - 2x_Ax_B$

$AB^2 = S^2 - 2P - 2P = (-2m-1)^2 - 4(5-2m) = 4m^2 + 4m + 1 - 20 + 8m = 4m^2 + 12m - 19$

$R = \frac{AB}{2} = \frac{\sqrt{4m^2+12m-19}}{2}$

$$(x + 2m - 1)^2 + y^2 = \frac{4m^2 + 12m - 19}{4}$$

$$(x + 2m - 1)^2 + y^2 = m^2 + 3m - \frac{19}{4}$$

(1)

7) Eqtion de la tangente : $y - y_E = a(x - x_E)$

$y + 5 = a(x - 2)$; $\boxed{y - ax + 2a + 5 = 0}$

$x^2 + y^2 - 6x - 4y + 3 = 0$

$(x-3)^2 - 9 + (y-2)^2 - 4 + 3 = 0$

$(x-3)^2 + (y-2)^2 = 10$ $\quad I(3,2) \quad R = \sqrt{10}$

$d(I,(d)) = R \longrightarrow \dfrac{|-3a + 2 + 2a + 5|}{\sqrt{1 + a^2}} = \sqrt{10}$

$|-a + 7| = \sqrt{10(1+a^2)}$

$a^2 - 14a + 49 - 10 - 10a^2 = 0$ $\quad \boxed{-9a^2 - 14a + 39 = 0}$

$a_1 = \dfrac{13}{9}$, $a_2 = -3$

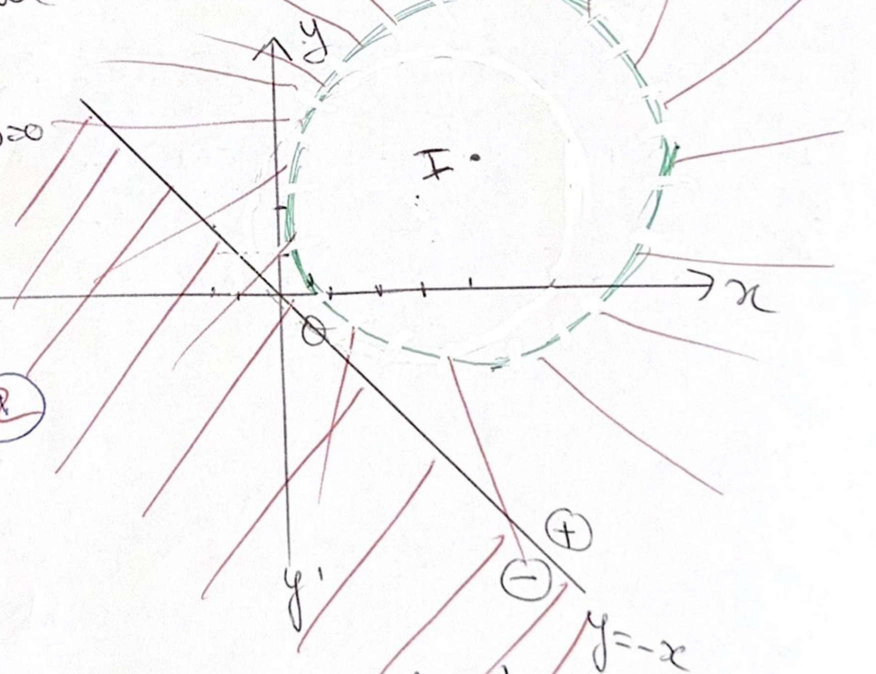
$(T_1) : y - \dfrac{13}{9}x + \dfrac{71}{9} = 0$ $\quad$ 1,5

$(T_2) : y + 3x - 1 = 0$

8) $x^2 + y^2 - 6x - 4y + 3 < 0$

Intérieur du cercle $\mathscr{C}$ $(I(3,2)$ et $R = \sqrt{10})$

$\begin{cases} y = -x \\ x^2 + \cancel{y}x^2 - 6x + 4x + 3 = 0 \\ 2x^2 - 2x + 3 = 0 \\ \text{pas de solution} \end{cases}$



$\begin{cases} x + y \geq 0 \\ 1 + 1 \geq 0 \\ 2 \geq 0 \end{cases}$

La solution est l'ensemble des points situés dans la partie non hachurée du repère.